Working on the Montague Grammar

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Our works

北陸先端科学技術大学院大学 先端科学技術研究科
先端科学技術専攻 情報科学分野 知能ロボティクス領域
東条敏研究室

- Linguistics
  - Text analysis (Novel, Ligal documents etc)
  - Agent communication
  - Dynamic Epistemic Logic
  - Multi-valued Logic
- Music
  - What is music?
  - Does music have the semantics?
  - Finding grammars of Music
  - Iterated Learning Model

Question

Do you understand this sentence?

- ポチ ガ イキガ ヲ カムン

Question

Do you understand this sentence?

- ポチ ガ イキガ ヲ カムン
- ぽち が おとこ を かむ

Question

Do you understand this sentence?

- ポチ ガ イキガ ヲ カムン
- ぽち が おとこ を かむ
- Pochi bites the man
- Pochi dog bites a man

Who are bitted by Pochi? Let’s think about this sentence with **LOGIC**.

Background

Richard Montague introduce the foundation of the formal semantics to the linguistic in **The Proper Treatment of Quantification in Ordinary English (PTQ)**. And some liguists improve this theory with modal operators and find the new Grammar after PTQ.

Richard Montague earned Ph.D.in Philosophy under the direction of Alfred Tarski. **Contributions to the Axiomatic Foundations of Set Theory**

Goal

The aim of this paper is to present in a regorous way the syntax and semantics of a certain fragment of a certain dialect of English. (Richard Montague, 1973)

1. Natural Language
2. Syntax of Linguistics (CCG)
3. Syntax of Logic (Intensional Logic = Modal Logic + Typed Lambda Calculus)
4. Semantics of Logic (Intensional Logic)

The Syntax of a Fragment of English

Combinatory Categorial Grammar

Definition (Combinatory Categorical Grammars)
A ccg is a five tuple $G = (V_N, V_T, S, f, R)$ where

- V_N is a finite set of nonterminals (atomic categories),
- V_T is a finite set of terminals (lexical items),
- S is a distinguished member of V_N,
- f is a function that maps elements of V_T to finite subsets of cat(V_N) and
- R is a finite subset of $\mathcal{C}(V_N)$

Combinatory Categorial Grammar

Where $cat(V_N)$ is defined as the smallest set of:

- $V_N \subset cat(V_N)$
- if c_1 ans c_2 are categories in $cat(V_N)$ then (c_1/c_n) and $(c_1\backslash c_2)$ are in $cat(V_N)$

and $\mathcal{C}(V_N)$ is defined as combinatory rules of V_N. *e.g.*,

$$x/y \; y \rightarrow x$$

$$y \; x\backslash y \rightarrow x$$

Example of Combinatory Categorial Grammar

1. $V_N := \{t, e, \dots\}$
2. $V_T := \{a, the, man, pochi, bites, sleeps, \dots\}$
3. $S := \{t\}$
4. f is a map which,
    - $a \in V_T \mapsto \{(t/(t/e))/(t/e), \dots\} \in \mathcal{P}(cat(V_N))$
    - $the \in V_T \mapsto \{(t/(t/e))/(t/e), \dots\} \in \mathcal{P}(cat(V_N))$
    - $man \in V_T \mapsto \{t/e, \dots\} \in \mathcal{P}(cat(V_N))$
    - $pochi \in V_T \mapsto \{t/(t/e), \dots\} \in \mathcal{P}(cat(V_N))$
    - $bites \in V_T \mapsto \{(t/e)/(t/(t/e)), \dots\} \in \mathcal{P}(cat(V_N))$
    - $sleeps \in V_T \mapsto \{t/e, \dots\} \in \mathcal{P}(cat(V_N))$
5. $R := \{x\ x\backslash y \to x,\ x/y\ y \to x, \dots\}$

Parse tree with CCG

"Pochi bites a man" is parsed as follows:

$$\dfrac{\text{Pochi} : t/(t/e) \qquad \dfrac{\text{bites} : (t/e)/(t/(t/e)) \qquad \dfrac{\text{the} : (t/(t/e))/(t/e) \qquad \text{man} : t/e}{\text{a man} : t/(t/e)}}{\text{bites the man} : t/e}}{\text{Pochi bites a man} : t}$$

Context Free Grammar

Context Free Grammar

- S · · · Sentense
- NP · · · Noun Phrase
- N · · · Noun
- VP · · · Verb Phrase
- TV · · · Transitive Verb
- IV · · · Intransitive Verb
- Det · · · Article

Context Free Grammar

- $S \rightarrow NP\, VP$
- $NP \rightarrow Det\, N$
- $VP \rightarrow TV\, NP$
- $VP \rightarrow IV$

- $Det \rightarrow a$
- $Det \rightarrow the$
- $N \rightarrow man$
- $N \rightarrow dog$
- $TV \rightarrow bite$
- $IV \rightarrow sleep$

Intensional Logic

Syntax

Syntax

- Constants: $c_0, c_1, c_2, \ldots$
- Variables: $x_0, x_1, x_2, \ldots$
- Predicates: $P_0, P_1, P_3, \ldots$
- Binary identity predicate: $=$
- Connectives: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
- Quantifiers: $\forall, \exists$
- Modal, temporal operators: $\Box, \mathbf{P}, \mathbf{F}$
- Brackets: $[,], (,)$
- Special Operators: $^\wedge, ^\vee, \lambda x[\ldots x \ldots]$

Syntax

Let $t_0, t_1, \ldots$ be constants and variables, and let $f_1, f_2, \ldots$ be formulae, and x be a variable. Then

- $P(t_0, t_1, \ldots, t_n)$ and $t_0 = t_1$ are formulae.
- $\neg f_0$ and $f_0 \wedge f_1$, $f_0 \vee f_1$, $f_0 \rightarrow f_1$, $f_0 \leftrightarrow f_1$ are formulae.
- $\forall x f_0$ and $\exists x f_0$ are formulae.
- $\Box f_0$ and $\Diamond f_0$, $\mathbf{P} f_0$, $\mathbf{F} f_0$ are formulae.
- ${}^\wedge f_0, {}^\vee f_0, \lambda x[f_0]$ are formulae

Intension and Extension

Pochi bites a man.

1. Pochi bites who is male.
2. Pochi bites who is a man walking on the street.

1 is **extensional** understanding. ($^\vee{}^\wedge a$)
2 is **intensional** understanding. ($^\wedge a$)

Typed Lambda Calculus

We translate categories to types of the lambda calculus

- t is t
- e is e
- t/e is $\langle e, t \rangle$

e.g.,

$$\text{Pochi} : t/(t/e) \iff \lambda P[{}^{\vee}P(pochi)] : \langle\langle s, \langle e, t\rangle\rangle, t\rangle$$

$$\text{bites} : (t/e)/(t/(t/e))$$

$$\iff \lambda x \lambda y[bites(x)(y)] : \langle\langle s, \langle\langle s, \langle e, t\rangle\rangle, t\rangle\rangle, \langle e, t\rangle\rangle$$

$$\text{a} : (t/(t/e))/(t/e)$$

$$\iff \lambda P \lambda Q \exists x[{}^{\vee}P(x) \wedge {}^{\vee}Q(x)] : \langle\langle s, \langle e, t\rangle\rangle, \langle\langle s, \langle e, t\rangle\rangle, t\rangle\rangle$$

$$\text{man} : t/e \iff \lambda x[man(x)] : \langle e, t\rangle$$

Parse Tree in Typed Lambda Calculus

Intensional understanding

$$
\dfrac{\lambda P[{}^{\vee}P(pochi)] : \langle\langle s, \langle e,t\rangle\rangle, t\rangle \qquad \dfrac{\lambda x \lambda y[bites(x)(y)] : \langle\langle s, \langle\langle s, \langle e,t\rangle\rangle, t\rangle\rangle, \langle e,t\rangle\rangle \qquad \dfrac{\lambda P \lambda Q \exists x[{}^{\vee}P(x) \wedge {}^{\vee}Q(x)] : \langle\langle s, \langle e,t\rangle\rangle, \langle\langle s, \langle e,t\rangle\rangle, t\rangle\rangle \qquad \lambda x[man(x)] : \langle e,t\rangle}{\lambda Q \exists x[{}^{\vee\wedge}man(x) \wedge {}^{\vee}Q(x)] : \langle\langle s, \langle e,t\rangle\rangle, t\rangle}\ {}^{\wedge}\beta}{\lambda y[bites({}^{\wedge}\lambda Q \exists x[{}^{\vee\wedge}man(x) \wedge {}^{\vee}Q(x)])(y)] : \langle e,t\rangle}\ {}^{\wedge}\beta}{bites({}^{\wedge}\lambda Q \exists x[{}^{\vee\wedge}man(x) \wedge {}^{\vee}Q(x)])(pochi) : t}\ {}^{\wedge}\beta
$$

Parse Tree in Typed Lambda Calculus

extensional understanding

$$
\dfrac{
\dfrac{\lambda P \lambda Q \exists x[{}^{\vee}P(x) \wedge {}^{\vee}Q(x)] : \langle\langle s, \langle e,t\rangle\rangle, \langle\langle s,\langle e,t\rangle\rangle, t\rangle\rangle \qquad \lambda x[man(x)] : \langle e,t\rangle}{\lambda Q \exists x[{}^{\vee\wedge}man(x) \wedge {}^{\vee}Q(x)] : \langle\langle s,\langle e,t\rangle\rangle, t\rangle}\ {}^{\wedge}\beta
\qquad
\dfrac{
\dfrac{\lambda P[{}^{\vee}P(pochi)] : \langle\langle s,\langle e,t\rangle\rangle,t\rangle \qquad \dfrac{\lambda x \lambda y[bites(x)(y)] : \langle\langle s,\langle\langle s,\langle e,t\rangle\rangle,t\rangle\rangle,\langle e,t\rangle\rangle \qquad \lambda P[{}^{\vee}P(x_0)] : \langle s,\langle\langle s,\langle e,t\rangle\rangle,t\rangle\rangle}{\lambda y[bites({}^{\wedge}\lambda P[{}^{\vee}P(x_0)])(y)] \quad \langle e,t\rangle}\ {}^{\wedge}\beta}{bites({}^{\wedge}\lambda P[{}^{\vee}P(x_0)])(pochi) \quad t}\ {}^{\wedge}\beta
}{\lambda x[bites({}^{\wedge}\lambda P[{}^{\vee}P(x)])(pochi)] : \langle e,t\rangle}
}{\exists x[{}^{\vee\wedge}man(x) \wedge bites({}^{\wedge}\lambda P[{}^{\vee}P(x)])(pochi)] : t}\ {}^{\wedge}\beta
$$

Usage of $\Box, \forall, \exists$

- neccesary: $\lambda p[\Box {}^\wedge p] : \langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle$
- every: $\lambda Q \lambda P \forall x[Q(x) \rightarrow P(x)] : \langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle$
- some: $\lambda Q \lambda P \exists x[Q(x) \wedge P(x)] : \langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle$

Semantics

Model

A models is 5-tuple $\langle W, I, <, U, V\rangle$.

- W is a set of worlds
- I is a set of instants
- U is a domain of individuals.
- V is a set of assignment functions about intensions
    - $V(c_0)(w, i) \in U$
    - $V(P_0)(w, i) \subset U^n$, n is number of arguments of P_0.

Semantics

- $[\![c_0]\!]^{M,w,i,g} = V(c_0)(i,w)$
- $[\![x_0]\!]^{M,w,i,g} = g(x_0)$, g gives extensional value of x_0.
- $[\![P_0(t_1, t_2, \ldots, t_n)]\!]^{M,w,i,g} = 1$
  $\iff \langle [\![t_0]\!]^{M,w,i,g}, [\![t_0]\!]^{M,w,i,g}, \ldots, [\![t_0]\!]^{M,w,i,g} \rangle \in [\![P_0]\!]^{M,w,i,g}$
- $[\![\neg f_0]\!]^{M,w,i,g} = 1 - [\![f_0]\!]^{M,w,i,g}$,
  $[\![f_0 \wedge f_1]\!]^{M,w,i,g} = \min([\![f_0]\!]^{M,w,i,g}, [\![f_1]\!]^{M,w,i,g})$,
  $[\![f_0 \vee f_1]\!]^{M,w,i,g} = \max([\![f_0]\!]^{M,w,i,g}, [\![f_1]\!]^{M,w,i,g})$,
  $[\![f_0 \rightarrow f_1]\!]^{M,w,i,g} = \max(1 - [\![f_0]\!]^{M,w,i,g}, [\![f_1]\!]^{M,w,i,g})$,
  $[\![f_0 \leftrightarrow f_1]\!]^{M,w,i,g} = [\![f_0 \rightarrow f_1 \wedge f_1 \rightarrow f_0]\!]^{M,w,i,g}$
- $[\![\forall x f_0]\!]^{M,w,i,g} = \min_x([\![f_0]\!]^{M,w,i,g})$,
  $[\![\exists x f_0]\!]^{M,w,i,g} = \max_x([\![f_0]\!]^{M,w,i,g})$

Semantics

- $[\![\Box f_0]\!]^{M,w,i,g} = \min_{w',i'}([\![f_0]\!]^{M,w',i',g})$,
  $[\![\mathbf{P} f_0]\!]^{M,w,i,g} = \min_{i'<i}([\![f_0]\!]^{M,w,i',g})$,
  $[\![\mathbf{F} f_0]\!]^{M,w,i,g} = \min_{i<i'}([\![f_0]\!]^{M,w,i',g})$
- $[\![{}^\wedge f_0]\!]^{M,w,i,g} = (w', i') \mapsto [\![f_0]\!]^{M,w',i',g}$
- $[\![{}^\vee f_0]\!]^{M,w,i,g} = [\![f_0]\!]^{M,w,i,g}(w, i)$

e.g., $[\![bites({}^\wedge \lambda Q \exists x[{}^{\vee\wedge} man(x) \wedge {}^\vee Q(x)])(pochi)]\!]^{M,w,i,g} = 0/1$

Conclusion

Conclusion

$$\begin{aligned} \text{NL Exp.} &\iff \text{FL Exp.} \\ \text{Category} &\iff \text{Type} \\ \text{Sentence} &\iff \text{Proposition} \end{aligned}$$

We get the translation from Natural Language to IL
and understand the semantics of Natural Language via IL
but it isn't enough. If you know interesting expression in Natural Language, please tell me. Thank you!

Reference

1. Tojo, Satoshi. Gengo chishiki shinnen no ronri. Tōkyo: Omusha, 2006. Print.
2. Dowty, David R., Robert E. Wall, and Stanley Peters. Introduction to Montague semantics. Dordrecht, Holland Boston Hingham, MA: D. Reidel Publishing Company,Sold and distributed in the U.S.A. and Canada by Kluwer Boston Inc, 1981. Print.